Chapter 4

Seasonal Analysis of Time Series

Seasonal analysis is an important consideration in sales forecasting for two reasons:

1. Analysis of data for periods shorter than one year must take into account normal seasonal influences before the net effect of such data can be determined.
2. Forecasting for any period less than a year requires an expressed or implied forecast for the seasonal pattern; i.e., the usual forecast assumes the seasonal pattern in the forecast period will be the same or an extension of a slowly changing past seasonal pattern.

4.1 Concept and Some Examples of Seasonal Patterns

Let us begin by describing the purpose of seasonal patterns through graphic examples. This approach provides insight into what to expect from a seasonal calculation and how to make a preliminary judgment as to whether the seasonal calculation is successful based on visual inspection of a chart.

The concept of seasonal factors depends on the recurrence of a pattern within years. The method for computing specific seasonal factors has four basic steps:

1. Calculate the historical ratios for a specific quarter or month to an annual moving average of quarters or months.
2. Form an array of all these ratios-to-moving average.
3. Discard or modify the extreme ratio-to-moving average values.
4. Average the remaining central ratios.

This four-step process results in a constant seasonal factor for a particular quarter or month and must be repeated for each of the four quarters or twelve months within a year. A further refinement consists of observing the time trend for the ratios and allowing slowly changing trends in the quarterly or monthly seasonal patterns to appear.

Statistical methodology for extracting seasonal factors depends on the four quarter or twelve month recurrence pattern. As there is no seasonal time-series component in annual observations, any consecutive annual data series contains no seasonal variation.

In this chapter we use quarterly data for most of the illustrations because quarterly data are the type most often encountered in sales forecasting having projections of economic variables as inputs. In addition, quarterly seasonal-adjustment patterns are easier to observe and assess than those for shorter monthly time intervals.

Sales for Safeway Stores Incorporated shown in Figure 4.1 illustrate a rather simple but typical quarterly pattern, namely, increased sales during the fourth quarter due to the seasonal influences of Thanksgiving and Christmas, and to a fiscal fourth quarter that is longer than the first three quarters. Notice that fourth quarter sales are appreciably and consistently higher than sales in either of the first three quarters in any year.

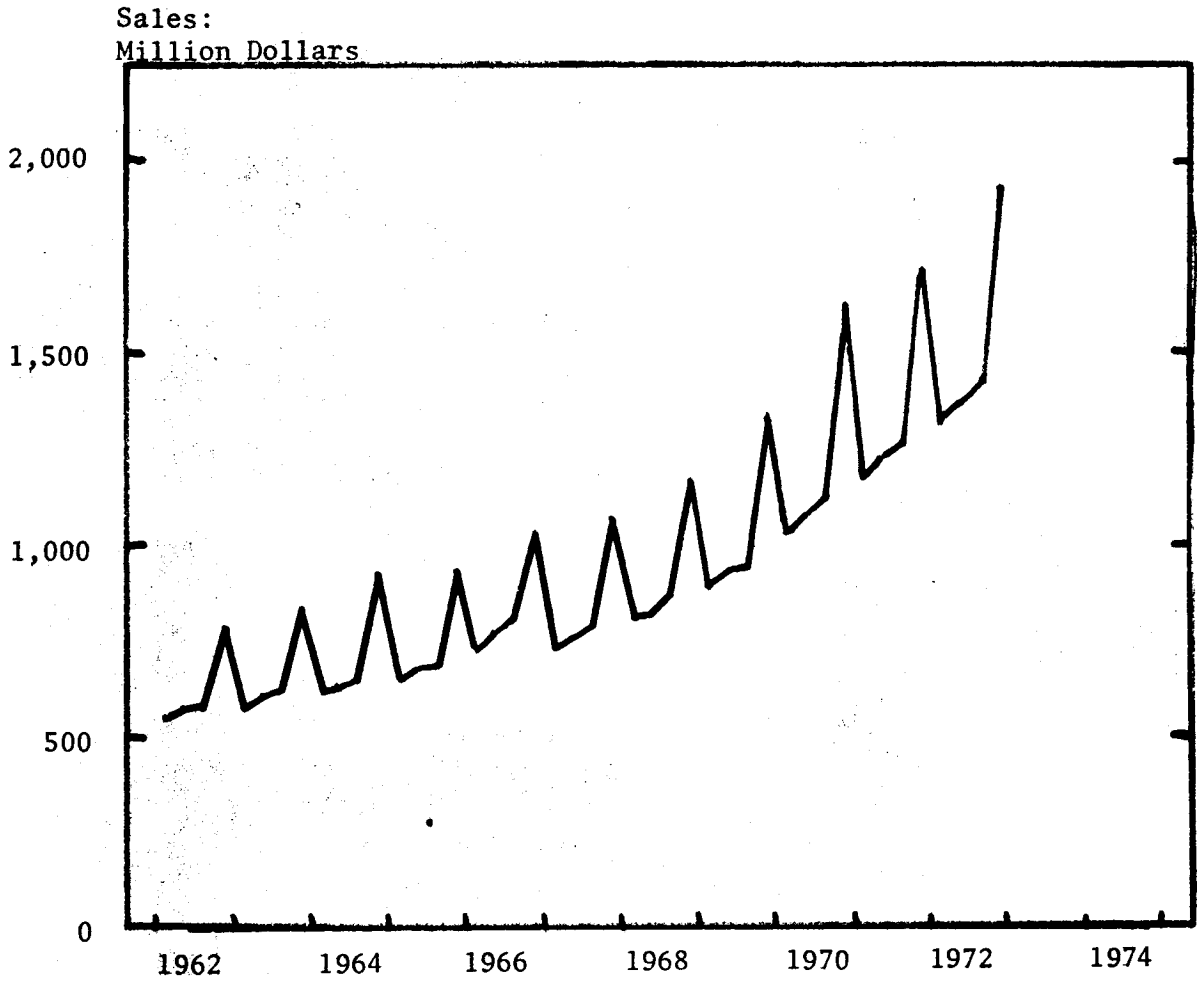
You will want to test your graphic analytical ability by determining from visual inspection that the upsurge in the fourth quarter is relatively stronger in recent years (1971-1972) than in earlier years (1962-1963). For example, fourth quarter 1972 appears on the chart to have an increase over third quarter 1972 that is more than twice as large as the increment of fourth quarter 1962 over third quarter 1962. But if we make rough mental calculations, we see that fourth quarter 1972 appears to be approximately the same percentage of its third quarter as a corresponding comparison for third and fourth quarters 1962. This characteristic is easier to analyze if we plot the same data on a semilogarithmic scale, as in Figure 4.2. Analysis of Figure 4.2 shows that the relative importance of the fourth quarter compared to the first, second, and third quarters is approximately the same in recent years as in earlier ones.[1]

Next, the seasonal pattern is calculated and expressed as a group of seasonal factors; that is, each quarter is expressed as a ratio of the average quarter (with no seasonal variation). The average quarter is one-fourth of a year and is arbitrarily set at 1.00. Listed here are the quarterly seasonal factors of retail sales for Safeway Stores Incorporated, computed on 1962 to 1972 data:

Figure 4.1

Safeway Stores Incorporated

Unadjusted Quarterly Sales, Current Dollars



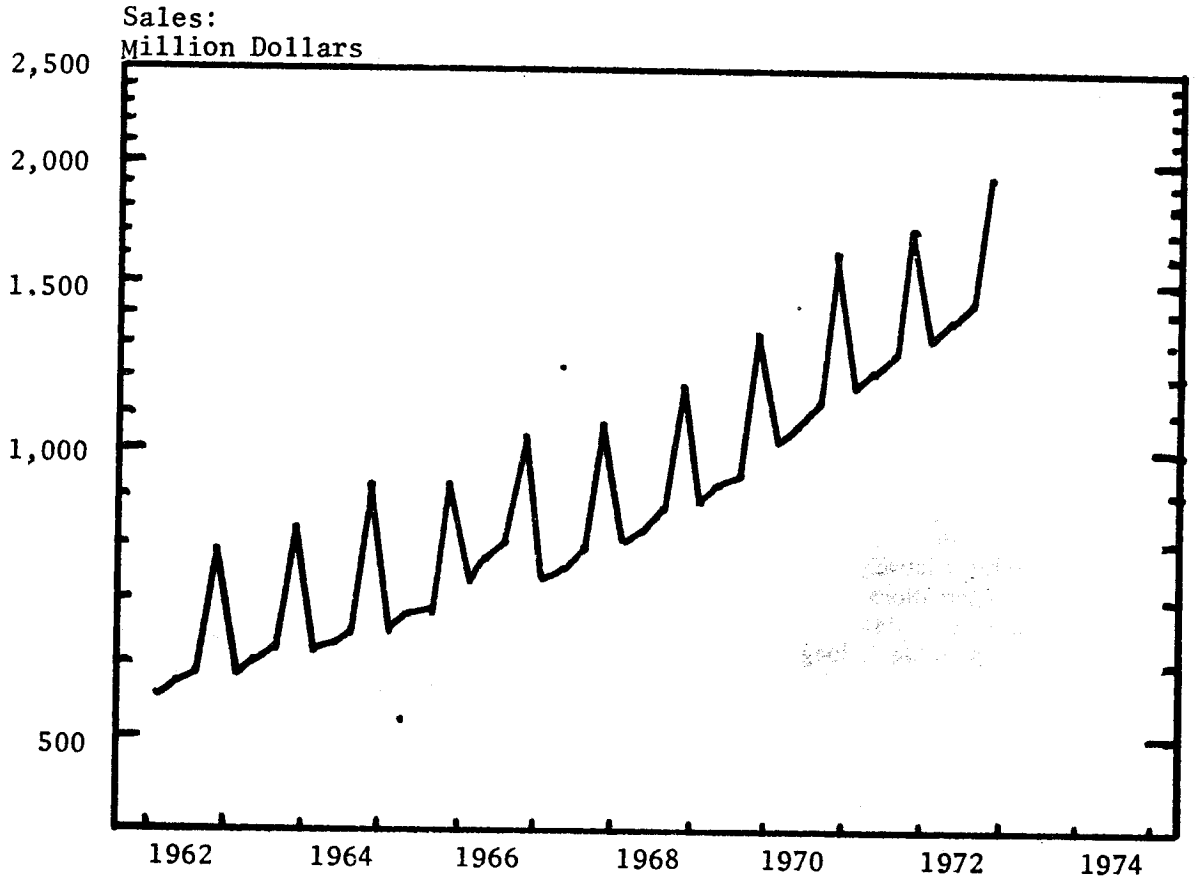
Source: Safeway Stores Incorporated

Figure 4.2

Safeway Stores Incorporated

Unadjusted Quarterly Sales, Current Dollars

Logarithmic Vertical Scale



Source: Safeway Stores Incorporated

Quarter	Seasonal Factor (Ratio to average quarter)
1	0.91
2	0.92
3	0.93
4	1.24

The seasonal adjustment procedure involves dividing original data by the respective seasonal factors. For example, $1,707 million original sales in fourth quarter 1971 divided by 1.24 (the seasonal factor for the fourth quarter) equals $1,377 million. This seasonally adjusted number means that sales would have been $1,377 million if the fourth quarter had been an *average* quarter rather than the seasonally high fourth quarter. Thus a seasonally adjusted series is one having no seasonal variation remaining in it.

The results of seasonally adjusting the Safeway data by an advanced ratio-to-moving average method with a changing seasonal pattern are shown in Figure 4.3. Careful examination of the figure reveals no recurring pattern within each year, and we conclude that this seasonal adjustment process is successful.

Notice that the seasonal pattern is fairly stable during the eleven-year history. In addition, the quarterly seasonally adjusted Safeway data give about the same visual impression as the annual data in Figure 3.5. When other kinds of time series with pronounced cyclical variations are encountered, however, the quarterly seasonally adjusted data greatly improve the ability to measure amplitudes and lengths of business cycles and major irregular fluctuations. This characteristic emphasizes that studies of business cycles, a critical ingredient of sales forecasting, must use at least seasonally adjusted quarterly data and, in some cases, seasonally adjusted monthly data in order to provide the earliest possible signal of a cyclical turning point. Our experience indicates that in most cases monthly data do not assist in this process because relatively random deviations often cause nonsignificant fluctuations in monthly data. On the other hand, quarterly data have the useful characteristics (1) of representing a long enough time period for economic observations to accumulate and (2) of being relatively undisturbed by random economic fluctuations within months. Seasonally adjusted monthly data, in many cases, must be smoothed to remove the minor irregular deviations, while quarterly totals automatically accomplish this end. This observation is confirmed by the predominant practice of using seasonally adjusted quarterly data in most econometric models.

Process Control Company's unadjusted quarterly sales, shown in Figure 4.4, reveal a less pronounced, but nonetheless observable, seasonal pattern than that for Safeway. Quarter two is the highest quarter in seven out of the twelve years for Process Control Sales, while quarter three is the lowest for nine of the years.

The seasonally adjusted sales in Figure 4.5 show an adequate adjustment for seasonal patterns, though the adjusted Process Control sales show much more irregularity than the Safeway sales (compare Figure 4.5 and 4.3). In assessing the adequacy of the seasonal adjustment in Figure 4.5, look for any recurring pattern of variation within years. Notice the oscillation pattern occurring in 1961-1964. Close examination of this pattern shows it to be an approximate three-quarter oscillation, rather than a four-quarter oscillation; this repetition is, therefore, not a recurring seasonal pattern. From 1965 on, the seasonally adjusted data follow cyclical peaks and troughs fairly consistently, leaving little reason to suspect existence of remaining seasonal pattern.

Notice also the substantial cyclical pattern for the Process Control sales exhibited in Figure 4.5 for 1965 through 1972 compared to the smooth trend of Safeway sales in Figure 4.3. This cyclical pattern in Process Control sales is important and could not have been perceived as well with annual data as with the seasonally adjusted quarterly data.

For Process Control the peak of 1966 occurs in quarter two, a fact visible in both the original data (Figure 4.4) and the seasonally adjusted data (Figure 4.5). The first quarter 1968 trough in Figure 4.5, however, is not readily apparent in Figure 4.4. In addition, the decline following the 1969 quarter-one peak is much steadier in the seasonally adjusted data than in the unadjusted data. Hence, these dips appear to be part of a persistent cyclical influence in the seasonally adjusted data.

4.2 Seasonal Adjustment by the Ratio-to-Moving Average Method

Calculation of a seasonal factor depends on representing annual data that contain no seasonal pattern as the sum of quarterly data for any continuous twelve-month period. The method starts, therefore, with the calculation of a four-quarter moving annual total which is converted into a moving average of four quarters. These moving averages are then centered at the middle of the quarterly data interval. Then the original, unadjusted quarterly figures are converted to ratios to the moving average. These ratios, called specific seasonal factors, next are arrayed in increasing order for each quarter; the extreme high and low ratios are discarded; and the central ratios-to-trend for each quarter constitute an approximate set of seasonal factors which must be normalized to average 1.00. At this point of the procedure attention is also directed to any changing seasonal influences and how they may be incorporated into calculations of seasonal factors.

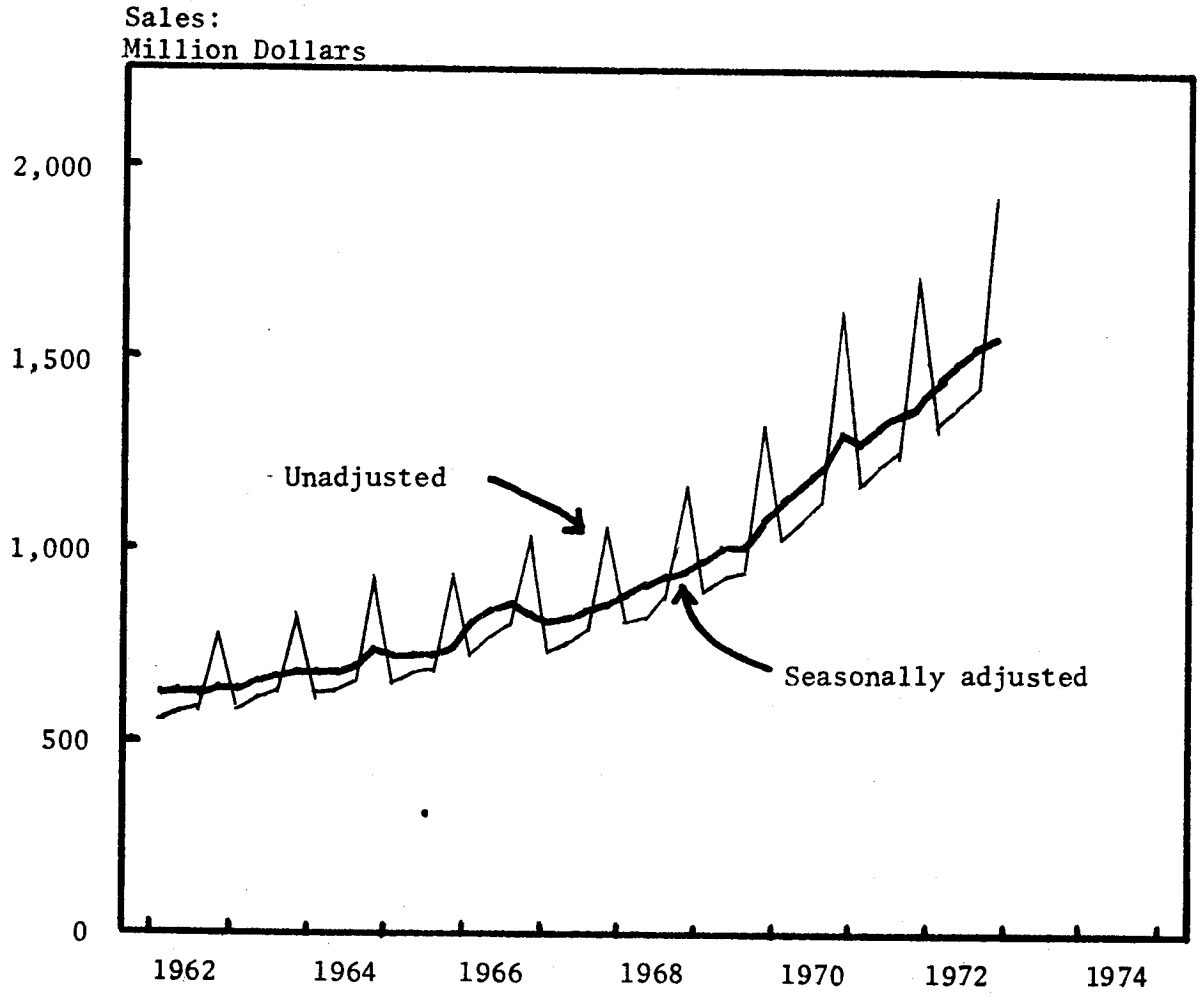
The detailed manual method of calculating seasonal factors for the Safeway data appear in Tables 4.1 and 4.2. Table 4.1 shows the original sales data in Column 2, the several steps in calculating a moving average, culminating in Column 5, and the resulting ratios-to-moving average in Column 6. The Column 6 figures are the "specific seasonal factors," where the word "specific" indicates that these are factors for particular quarters rather than average factors for all similar quarters in the time period. The process of centering the moving averages, as illustrated in Column 5, is required wherever an even number of periods (monthly or quarterly) is used within the year.

The moving average reported in Column 5 of Table 4.1 appears as the smooth line on Figure 4.6. The quarterly moving average, since it is based on an annual time period, has no seasonal pattern in it. The relationship between the

Figure 4.3

Safeway Stores Incorporated

Seasonally Adjusted Quarterly Sales, Current Dollars
Using X-11Q Computer Program

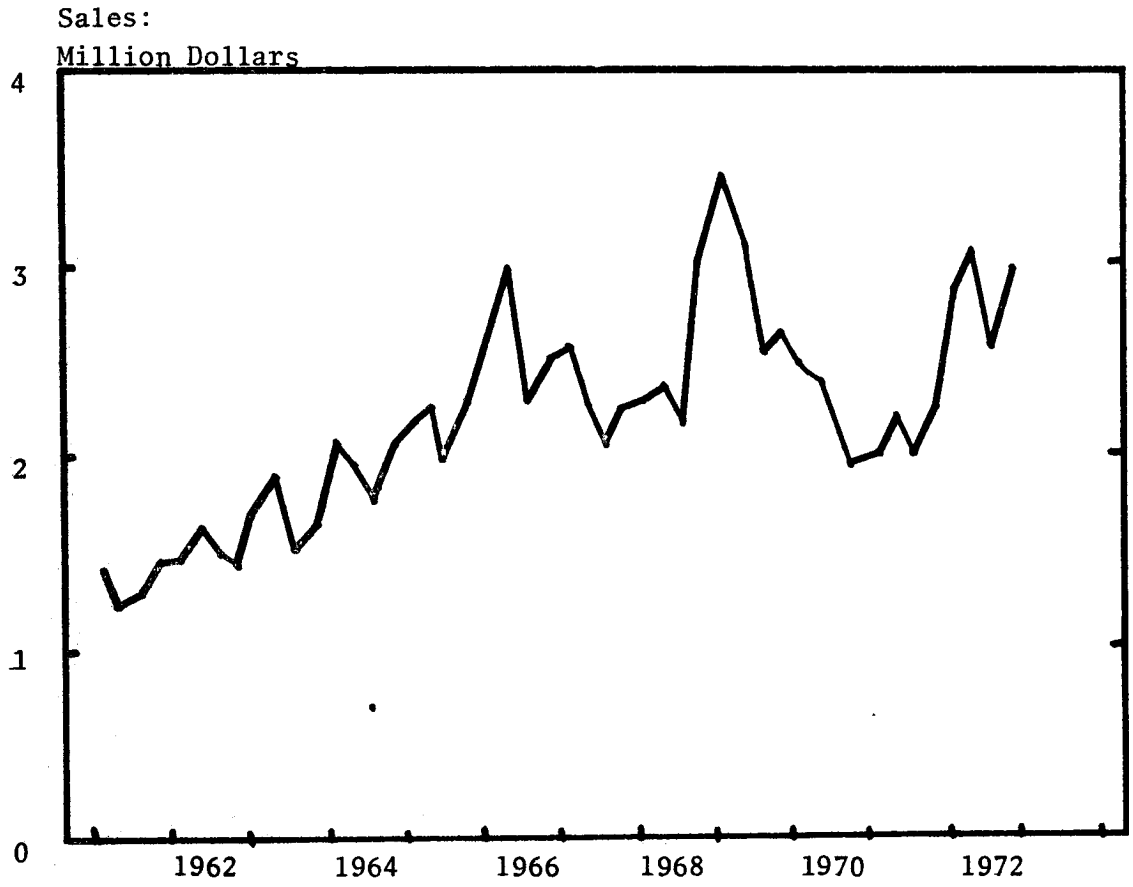


Source: Safeway Stores Incorporated

Figure 4.4

Process Control Company

Unadjusted Quarterly Sales, Current Dollars

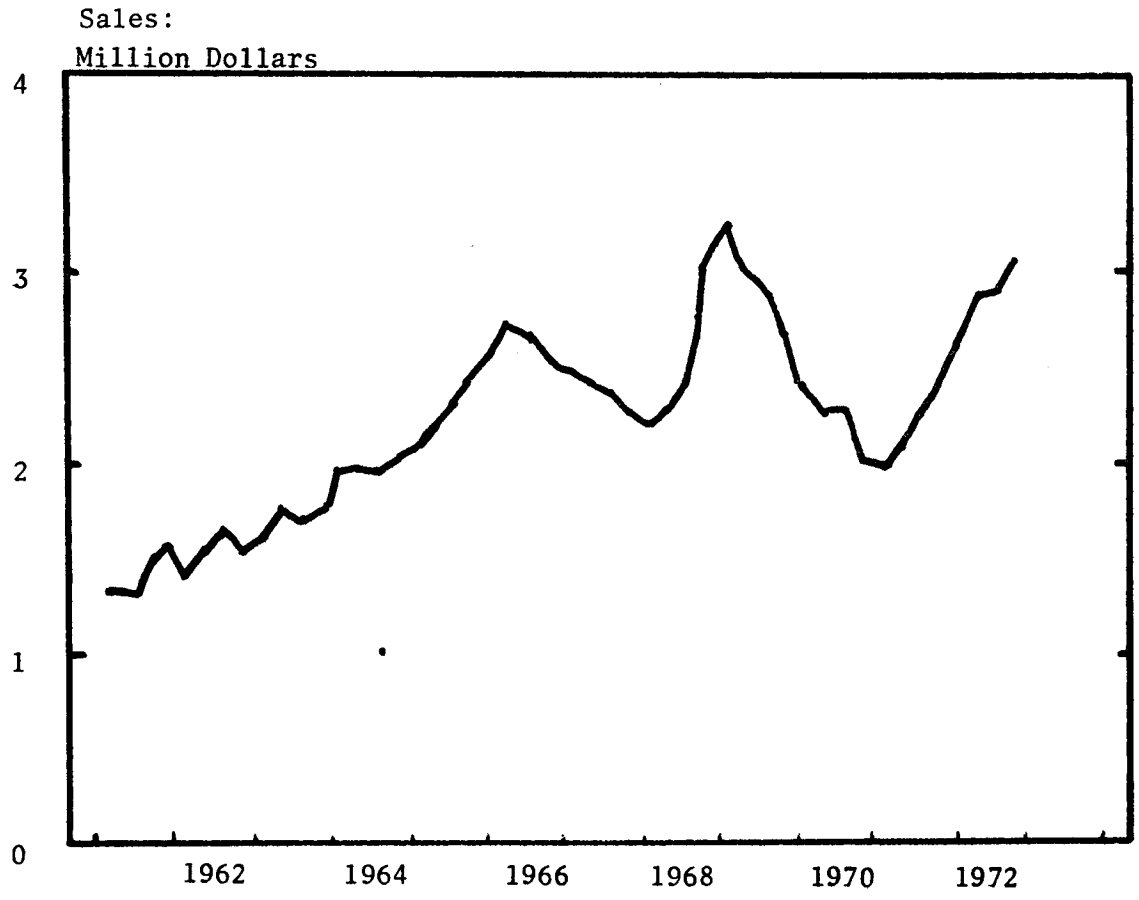


Source: Process Control Company

Figure 4.5

Process Control Company

Seasonally Adjusted Sales, Current Dollars



Source: Process Control Company

Table 4.1
Safeway Stores Incorporated
Seasonal Adjustment by Ratio-to-Moving Average Method

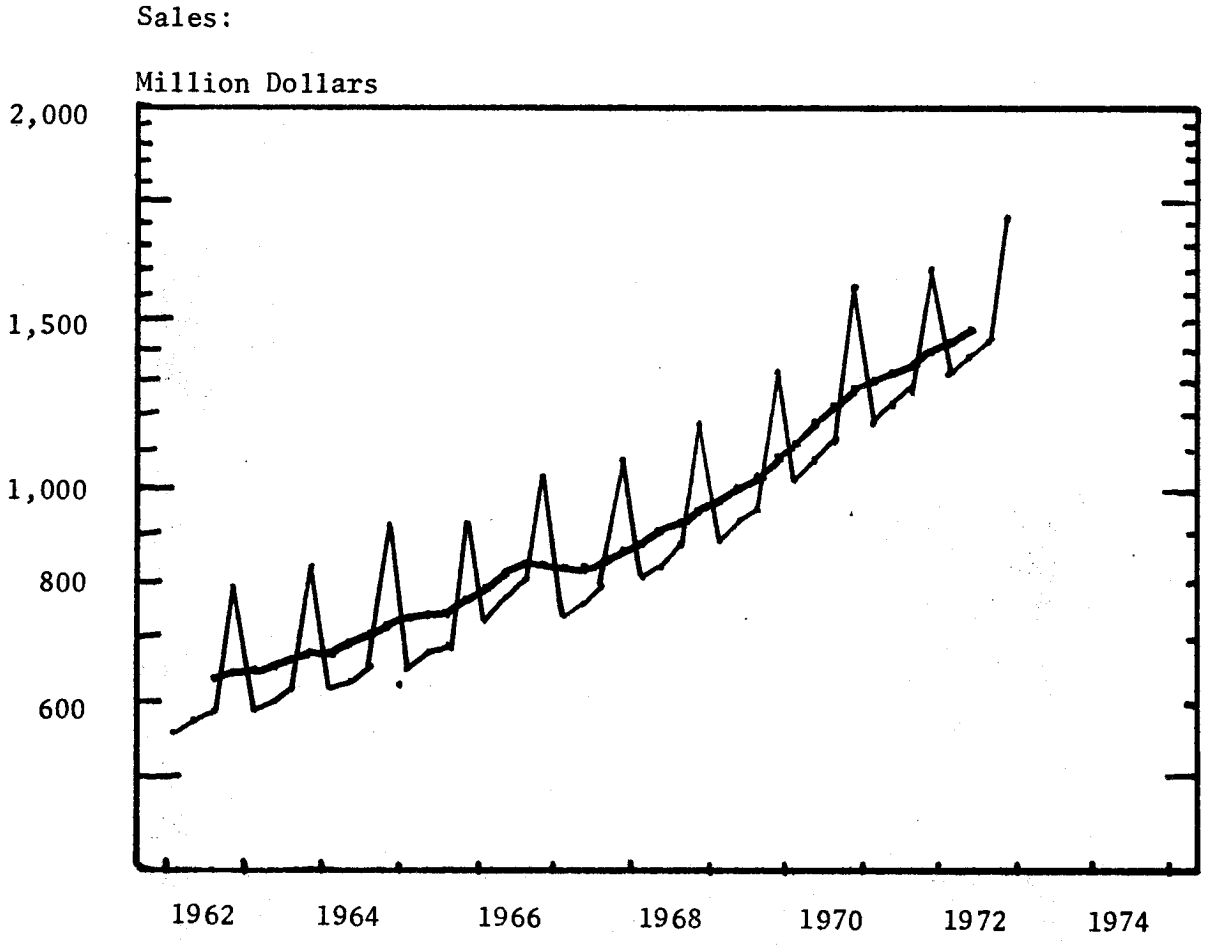
(1) Year and Quarter	(2) Sales	(3) 4-Quarter Moving Total (from (2))	(4) 4-Quarter Moving Average (3)÷4.0	(5) Quarterly Moving Average: Centered	(6) Ratio to Moving Average[a] (2)/(5)	(7) Constant Seasonal Factors (from Table 4.2, Part C, Line 4)	(8) Sales Seasonally Adjusted (2)/(7)
		------Million Dollars------			Ratio	Ratio	Mil. Dol.
1962-1	559					.911	614
2	576	2,510[b]	627.5			.920	626
3	586	2,538	634.5	631.0	.929	.930	630
4	789	2,567	641.8	638.1	1.236	1.239	637
1963-1	587	2,603	650.8	646.2	.908	.911	644
2	605	2,648[b]	662.0	655.4	.922	.920	658
3	622	2,679	669.8	665.9	.934	.930	669
4	834	2,701	675.2	672.5	1.240	1.239	673
			1964-1970 omitted				
1971-1	1,173	5,274	1,318.5	1,302.2	.901	.911	1,288
2	1,217	5,359[b]	1,339.8	1,329.2	.916	.920	1,323
3	1,261	5,512	1,378.0	1,358.9	.928	.930	1,356
4	1,708	5,670	1,417.5	1,397.8	1.222	1.239	1,378
1972-1	1,326	5,832	1,458.0	1,437.8	.922	.911	1,456
2	1,375	6,058[b]	1,514.5	1,486.2	.925	.920	1,495
3	1,423					.930	1,530
4	1,934					1.239	1,561

a = Specific seasonal factors
b = Calendar year annual total
Source: Safeway Stores Incorporated

Figure 4.6

Safeway Stores Incorporated

Unadjusted Quarterly Sales, Current Dollars
and
Four-Quarter Moving Average, Centered
On Logarithmic Vertical Scale



Source: Safeway Stores Incorporated

moving average and the unadjusted quarterly data is evident in this semilogarithmic chart, Figure 4.6, where the fourth quarter seasonal peaks are approximately a constant distance (hence, ratio on logarithmic scale) above the moving average and where the first, second, and third quarters are approximately a constant distance (ratio) below the moving average. This consistency of vertical distances as we have already indicated suggests the seasonal pattern to be calculated is reasonably constant.

The specific seasonal factors, or ratios-to-trend, for Safeway sales appear in Part A of Table 4.2, grouped for each quarter. Visual inspection of these data indicates very slight consistent change over time.

Part B of Table 4.2 shows the specific seasonal factors, by quarter, arrayed in order by size without regard to year. The process of calculating a constant seasonal factor involves discarding the highest and lowest third of the values. Then an arithmetic mean of the remaining one-third central values is calculated, as shown in lines 1 and 2 of Part C at the bottom of the table. The total of the four factors in line 2 is 3.9947 which should equal a total of 4.0000 if the average quarter is to equal 1.00. The normalizing process to accomplish this end is shown in line 3 of Part C, where each factor is multiplied by the ratio of 4.000/3.9947, or 1.00133. The resulting adjusted seasonal factors in line 3 total 4.00, so that the average quarter now equals the desired 1.00. This adjustment completes the calculation of constant seasonal factors (rounded off in line 4), the use of which presumes a constant or average seasonal pattern through the observed time period.

4.3 Constant vs. Changing Seasonal Patterns

To determine whether the seasonal factors have a consistent pattern of change over time, we plotted the specific seasonal factors by quarter (ratios-to-moving average) in Figure 4.7. Examination of the plots indicates slight irregularity but no consistent trend, upward or downward, for any of the four quarters. In the Safeway case, therefore, a constant seasonal pattern is justified, and no further calculation is necessary.

Figure 4.8 shows the seasonally adjusted Safeway data using constant seasonal factors. Within the accuracy discernable on the chart, there is no apparent remaining seasonal pattern. Residual seasonal influences, however, may show up after the seasonal adjustment process as periodic variation within a year. Such results could have two sources of explanation. First, human error in data collection, computation, and the like might cause perceived failure in the efforts to seasonally adjust a series. Second, remaining seasonal pattern might indicate the need to use *changing*[2] rather than *constant* seasonal factors in computing adjusted sales (see Figure 4.8a). We emphasize that both sources of explanation do not imply failure in the seasonal adjustment technique per se, but rather they indicate possible misapplications of a proven body of methodology.

A summary assessment, here, is that use of changing seasonal factors generally permits more accurate seasonal adjustment than constant factors, since seasonal adjustment at the end of the time period will be more accurate. Such accuracy is needed at the end of the historical time period more than at any other time because this point constitutes the base from which forecasts are made.

We have chosen to discuss computations for constant rather than changing seasonal factors to establish conceptual understanding only for simplicity's sake, recognizing the extensive use of computer programs at the applied level.

4.4 Using Seasonal Adjustment in Forecasting Sales

Converting past data into seasonally adjusted data, so that recent trends can be calculated from quarterly observations, is one major use of seasonal patterns in sales forecasting. Seasonal adjustment factors are also applied to quarterly trend projections to put seasonal influences back into the seasonally adjusted data.

4.4.1 Seasonally Adjusting (Deseasonalizing) Data

Seasonally adjusted sales are obtained by dividing the original series by the appropriate seasonal factors, as shown here:

$$\frac{\text{Original Sales}}{\text{Seasonal Factor}} = \begin{matrix}\text{Seasonally}\\ \text{Adjusted}\\ \text{Sales}\end{matrix} \qquad (4.1)$$

Deseasonalized sales is a statistical concept and not an amount that would be recorded in company sales records. One use of seasonally adjusted sales is to determine whether sales in a recently completed quarter changed by more than an amount which could be attributed to average seasonal changes. A second application is to use seasonally adjusted sales as the basis from which recent trends can be accurately computed.

4.4.2 Combining Quarterly Forecasts With Seasonal Adjustment Factors

Combining a seasonally adjusted forecast with the seasonal pattern is the reverse of the deseasonalization process. The procedure is accomplished easily by multiplying the quarterly seasonally adjusted forecasts by the seasonal adjustment factors, as shown here and in Table 4.3:

$$\begin{bmatrix}\text{Seasonally Adjusted}\\ \text{Projected Sales}\end{bmatrix} \times \begin{bmatrix}\text{Seasonal Factor}\end{bmatrix} = \begin{bmatrix}\text{Trend and}\\ \text{Seasonal}\\ \text{Projection}\end{bmatrix} \qquad (4.2)$$

This process will be illustrated in detail in Chapter 15 in Table 15.3.

As a variation of this process, it is sometimes necessary to combine seasonal adjustment factors with annual trend projections into quarterly projections. For this purpose, Equation 4.2 changes to this form:

$$\begin{bmatrix}\text{Quarterly}\\ \text{Trend}\\ \text{Projection}\end{bmatrix} \times \begin{bmatrix}\text{Seasonal}\\ \text{Factor}\end{bmatrix} = \begin{bmatrix}\text{Trend and}\\ \text{Seasonal}\\ \text{Projection}\end{bmatrix} \qquad (4.2a)$$

Table 4.2

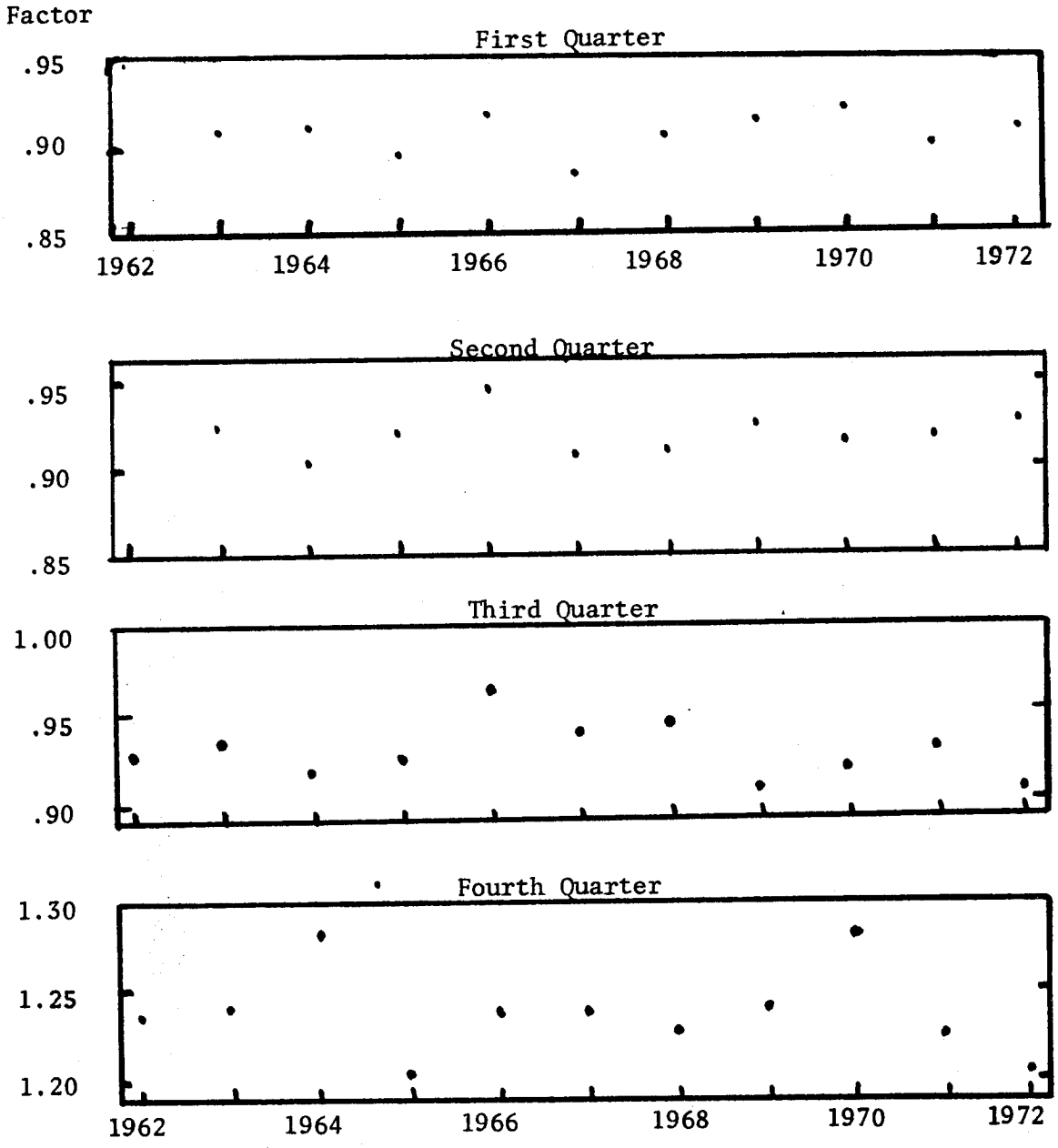
Safeway Stores Incorporated
Analysis of Ratios-to-Moving Average

Year	Quarter				
	1	2	3	4	
Part A: Ratios-to-Moving Average from Table 4.1					
1962	-	-	.929	1.236	
3	.908	.922	.934	1.240	
4	.910	.904	.918	1.281	
1965	.896	.919	.925	1.202	
6	.917	.944	.961	1.239	
7	.885	.906	.939	1.234	
8	.906	.919	.942	1.225	
9	.914	.923	.906	1.240	
1970	.921	.912	.917	1.280	
1	.901	.916	.928	1.222	
2	.922	.925			
Part B: Array of Ratios-to-Moving Average[a]					
	.922	.944	.961	1.281	
High ratios	.921	.925	.942	1.280	
	.917	.923	.939	1.240	
	.914	.922	.934	1.240	
Central ratios	.910	.919	.929	1.239	
	.908	.919	.928	1.236	
	.906	.916	.925	1.234	
	.901	.912	.918	1.225	
Low ratios	.896	.906	.917	1.220	
	.885	.904	.906	1.202	
Part C: Calculation of Seasonal Factors					
1. Sum of central ratios	3.638	3.676	3.716	4.949	
2. Average of central ratios[b]	.9095	.9190	.9290	1.2372	Annual sum 3.9947
3. Adjusted average = seasonal factors[c]	.9107	.9202	.9302	1.2389	4.0000
4. Rounded seasonal factors	.911	.920	.930	1.239	4.000

a/ From Part A of this table
b/ Modified mean = unadjusted seasonal factors
c/ Average of central ratios x 4.0000/3.9947 or 1.00133
Source: Table 4.1.

Figure 4.7

Time Plotting of Specific Quarterly Seasonal Factors

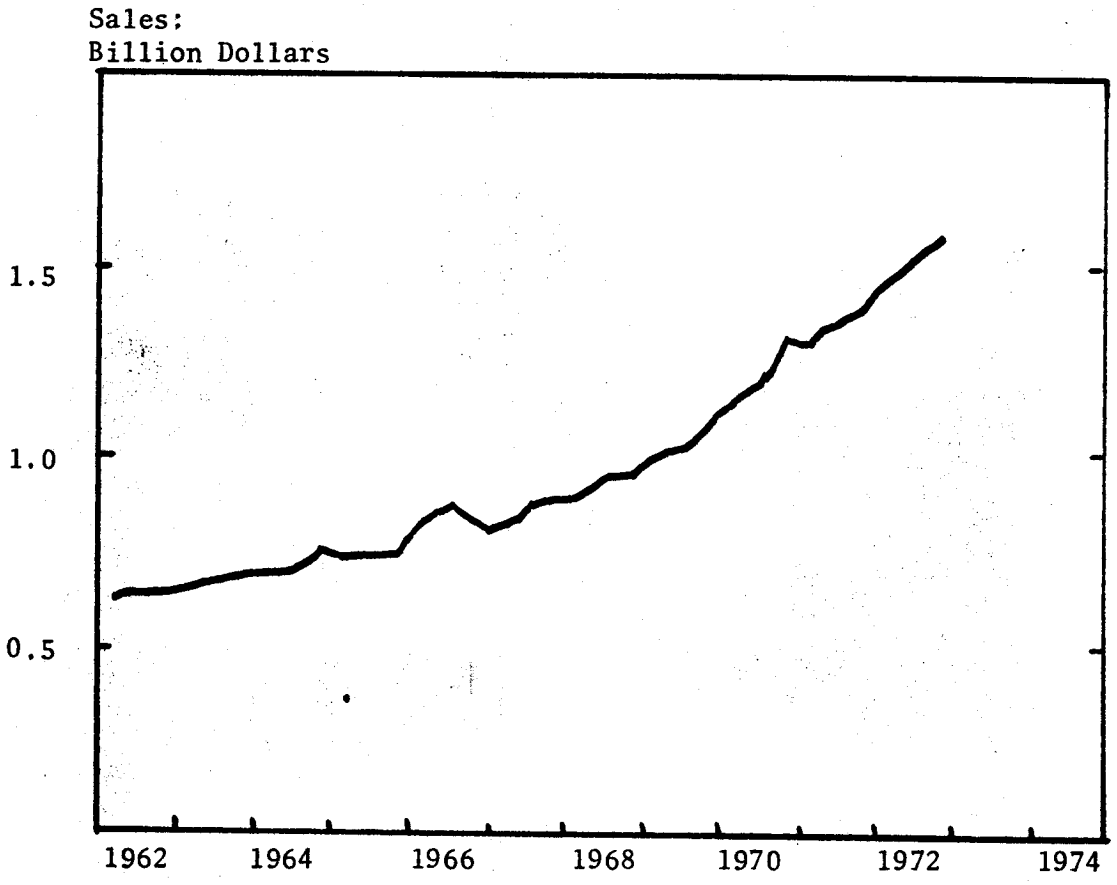


Source: Part A, Table 4.2

Figure 4.8

Safeway Stores Incorporated

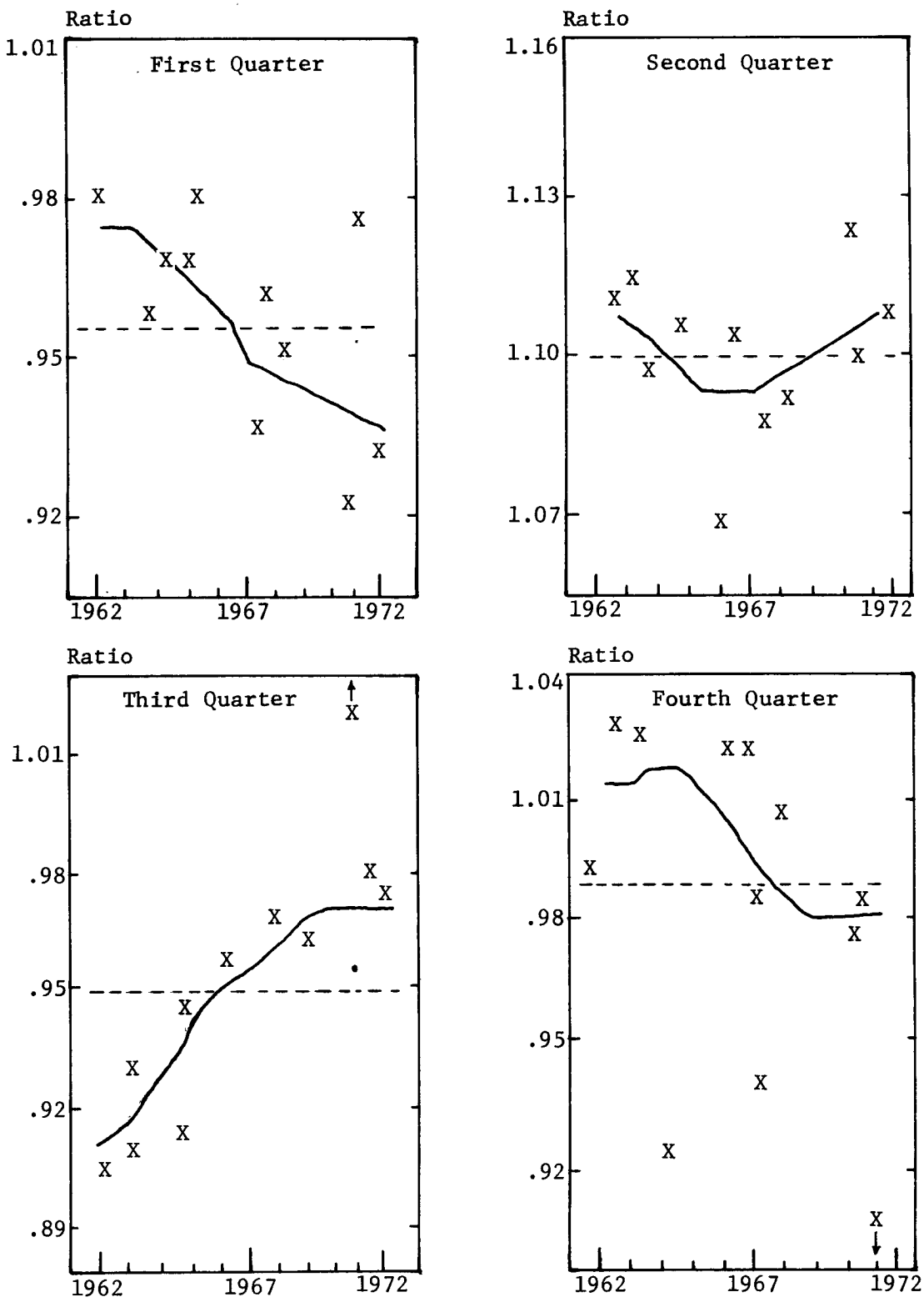
Seasonally Adjusted Quarterly Sales, Current Dollars



Source: Table 4.1, Column 8, using manually calculated constant seasonal factors from Table 4.2, Part C.

Figure 4.8a

Ratios-to-Moving-Average for Auto Dealer Retail Sales, and Changing Seasonal Factors



Source: X-11 Computer Program for Sales from Table 5

X = Ratios
——— = Changing Seasonal Factors
---- = Constant Seasonal Factors

Table 4.3

Safeway Stores Incorporated

Quarterly Trend Projections with Seasonal Adjustment Factors

(1) Year and quarter	(2) Quarterly trend projection	(3) Seasonal adjustment factors	(4) Quarterly trend and seasonal projection (2) x (3)
	bil. dol.		bil. dol.
1973-1	1.612	0.911	1.47
1973-2	1.652	0.920	1.52
1973-3	1.692	0.930	1.57
1973-4	1.732	1.239	2.15

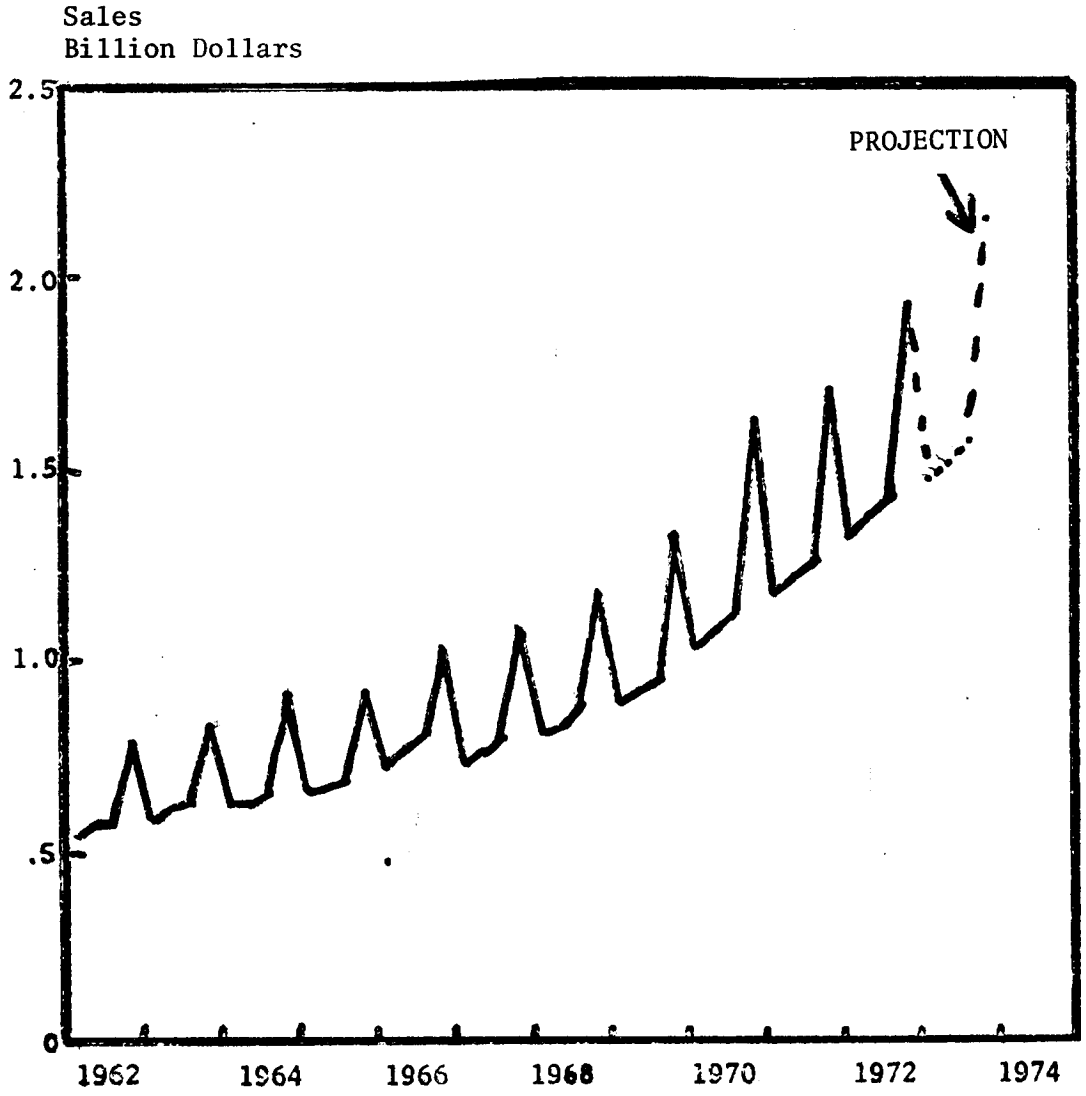
Source: Safeway Stores Incorporated and Table 4 .2, Part C.

Figure 4.9

Safeway Stores Incorporated

Quarterly Sales; Not Seasonally Adjusted and

1973 Linear Trend Projection Multiplied by Seasonal Factors



Source: Safeway Stores Incorporated and Section 4.3.3.

To accomplish this end, the projection of annual trend first must be converted to a projection of quarterly trend. This conversion process requires three arithmetic steps:

1. Convert the annual rate of change in annual data to an annual rate of change in quarterly data. Using data for Safeway for 1969-1972:
(2b) = $0.638 bil. = annual rate of change in annual data
(2b)/4 = $0.1595 bil. = annual rate of change in quarterly data.

2. Convert the annual rate of change in quarterly data to a quarterly rate of change in quarterly data:
(2b)/4 = $0.1595 bil. = annual rate of change in quarterly data
(2b)/4 x ¼ = $.0399 bil. = quarterly rate of change in quarterly data.

3. Change the annually centered trend value to a quarterly centered trend value. This result is accomplished by moving forward or backward from quarterly trend by one-half multiples of the quarterly rate of change.

Let us illustrate the complete conversion process for Safeway sales, 1973:

Annual trend for 1973 = $Y_{c(73)}$ = $6.690 billion

Quarterly trend for 1973 + 6.690/4 = $1.672 billion

Quarterly trend for 1973, 1st quarter = 1.672 – 2b/16 = 1.672 – .0399 = $1.632 billion

Quarterly trend for 1973, 2nd quarter = 1.672 – ½ x 2b/16 = 1.672 – ½(.0311) = 1.672 + .0200 = $1.6792 billion

Quarterly trend for 1973, 3rd quarter = 1.672 + ½ x 2b/16 = 1.672 = ½(.0399) = 1.672 + .0200 = $1.692 billion

Quarterly trend for 1973, 4th quarter = 1.692 + 2(½) x 2b/16 = 1.692 + .0399 = $1.732 billion

Notice that *2b* is the annual rate of change in this case because the "coded x" represents ½ year; that is, there are an even number of observations. If an odd number of observations were the case then "coded x" would represent one year, and the annual rate would be equal to *b*; the annual rate of change in quarterly data would be b/4; and the quarterly change in quarterly data would be b/16. The combination of quarterly trend projection (as a simple forecast) with seasonal factors is shown in Table 4.3.

4.4.3 Appraisal of Trend and Seasonal Projection as a Simple Forecast for Safeway

Plotting trend and seasonal projections as an extension of the original data helps in appraising the projections as preliminary forecasts. Figure 4.9 shows Safeway's historical seasonally adjusted quarterly data and the projection of trend and seasonal patterns for 1973. Since the annual trend fitted to the last four years, as shown in Figure 3.7, is a reasonably good fit and since the seasonal pattern is stable (see Figure 4.7), the combination projections of quarterly data are judged reasonable.

Given the past pattern of change in Safeway sales, namely that of fairly regular growth, this trend projection serves as a simple comparison forecast. It implies that the net balance of forces for change will continue at about the same rate in 1973 as in the last four years. Since Safeway has not announced any major speedup in mergers or acquisitions, but rather states in its 1972 annual report that upgrading of stores and discontinuance of old stores will continue, this projection may realistically describe the net internal forces for change. The external forces of population growth and slow changes in food consumption patterns will all tend to go forward by about the same rate. But inflation will probably be much faster in 1973 and, therefore, will probably provide a net upward force in current dollar sales.

Footnotes

1. The relative deviation of any extreme quarter is approximately measured on semilogarithmic paper by measuring vertical distances between points. Recall that vertical distances on a logarithmic scale represent ratios, rates, or percents of change. This useful characteristic of "ratio" chart is the basis for a method of graphically determining seasonal patterns. For more detail see Spurr, Kellogg, and Smith, *Business and Economic Statistics* (Homewood, Illinois, Richard D. Irwin, Inc., 1961), pp. 394-400.

2. Detailed calculations for changing seasonal factors are outside the purpose of this text but can be performed readily by using computer programs such as the X-11 Seasonal Adjustment Program of the Bureau of Census, U.S. Department of Commerce. See the *X-11 Variant of the Census Method II Seasonal Adjustment Program*, Technical Paper No. 15, 1967 revision.

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